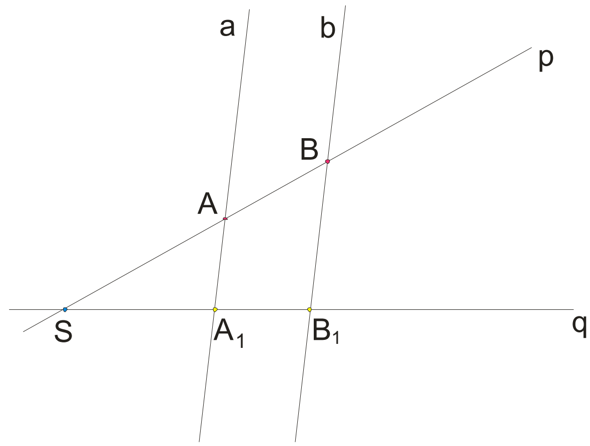
Thales' theorem

If parallel lines a and b intersect line p in real points A and B, and line in A_1 and B_1, and if S is a common point for lines p and q, then applies:

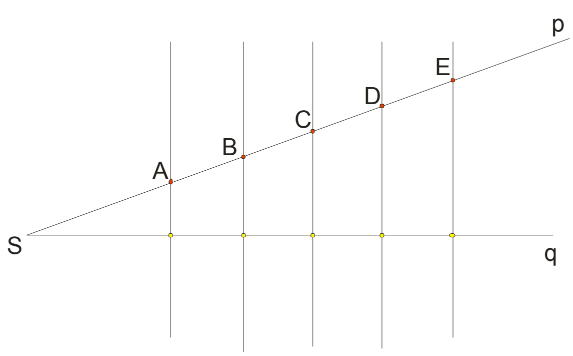
$$\frac{AA_1}{BB_1} = \frac{SA}{SB} = \frac{SA_1}{SB_1}$$

In picture this would look like this:

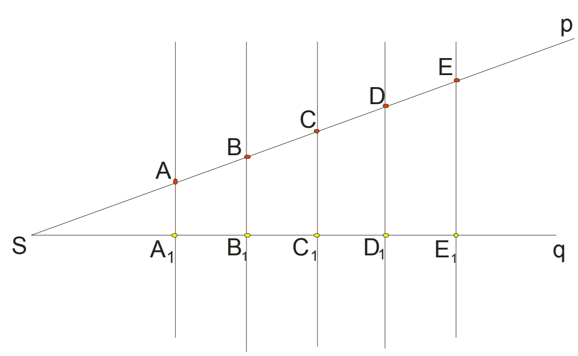


For Thales' theorem, we can have an important **conclusion:**

If two arbitrary lines p and q cuts series of parallel lines, so that the segments are equal among themselves, then the segments on the second line are mutually equal.



picture 1.



picture 2.

On **picture 1.** we have a series of parallel lines which make equal segments on Sp, $AB = BC = CD = DE$. Then the segments , by Thales' theorem, on Sq are also equal : $A_1B_1 = B_1C_1 = C_1D_1 = D_1E_1$ (picture 2.)

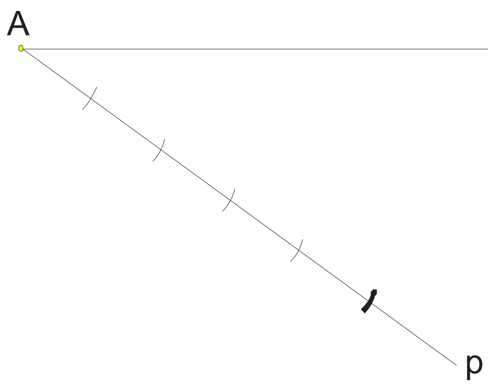
This conclusion is directly applicable in long division in equal parts.

Example 1. **Given along AB divided into five equal parts.**

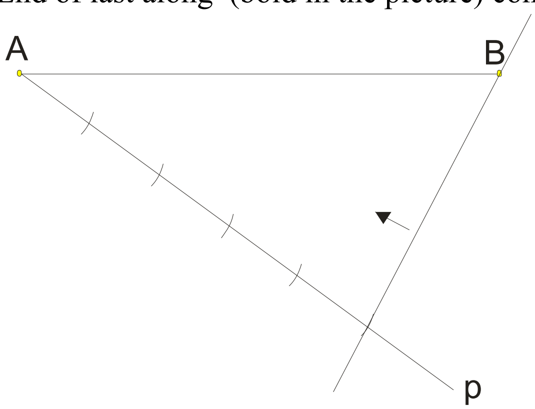
Solution

We take an arbitrary along AB:

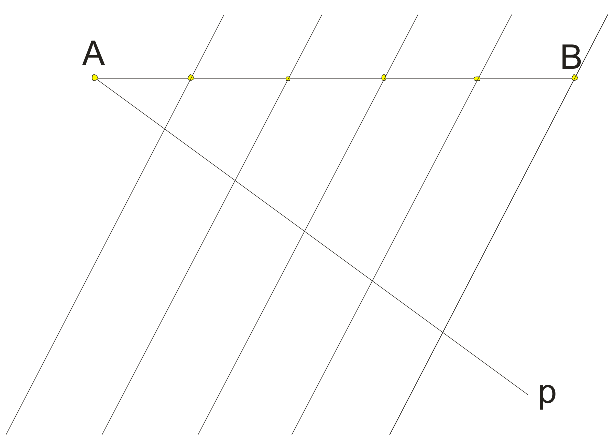
From point A draw line *Ap* (on either side). On it we draw five equal along.



End of last along (bold in the picture) connect with point B with line .



Parallel with this line we draw 4 more lines.



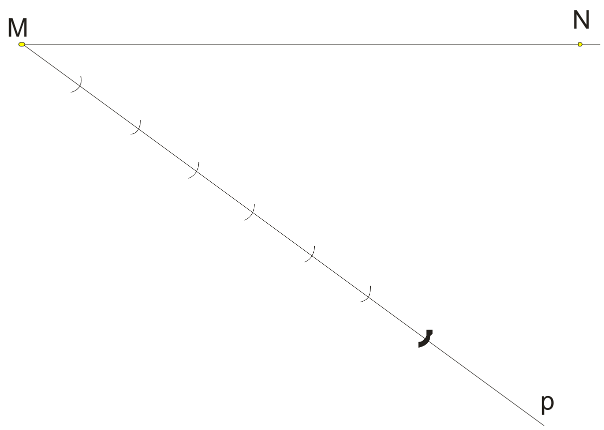
Given along Ab is divided on 5 equal parts!

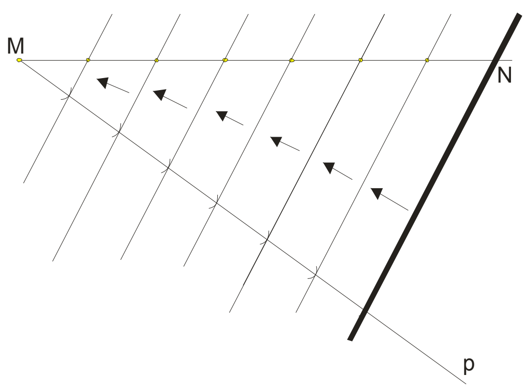
A similar procedure would be if we have to divid along in 3,4,6,7…parts.

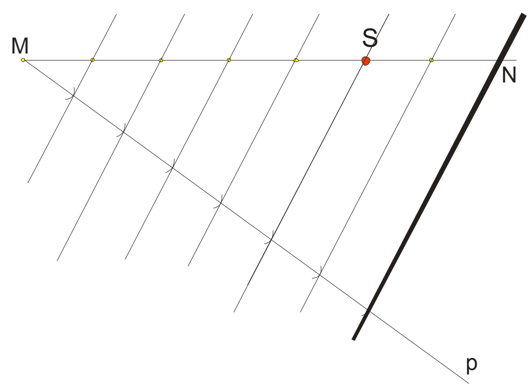
Example 2. **Given along MN divided in the ratio 5:2.**

Solution:

When we seek to divide along in a scale, we first gathered together all the parts :5+2=7. So, we share along at 7 equal parts:







Therefore, we divided along MN to 7 equal parts. Just count five and put the point, for example, S.

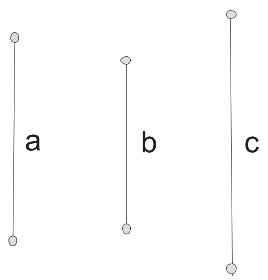
We are sure that: $MS:SN=5:2$

$\boxed{\text{Example 3.}}$ **Given an arbitrary longs *a, b* and *c*. Constructed along *x*, so that:** $a : b = c : x$

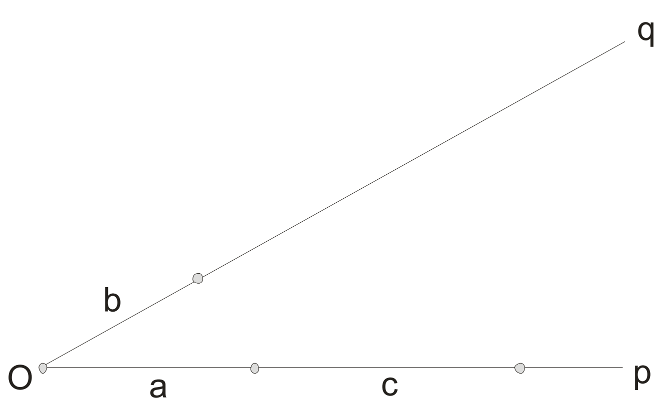
Solution

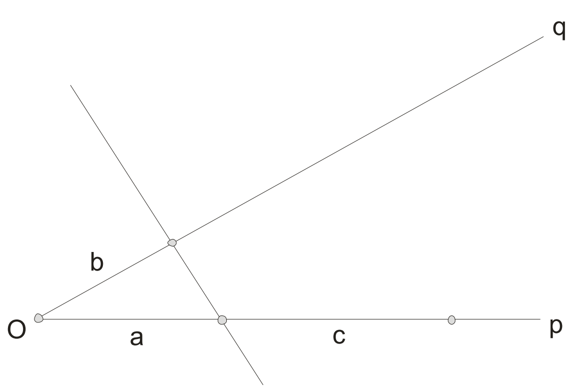
Here we will use Thales' theorem. Here it is important that x in the proportion, is in 4-th place. As we see , in this case it is satisfied.

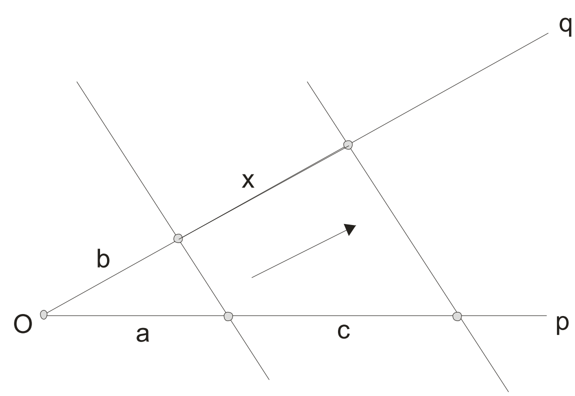
First, take three arbitrary long:



Draw an arbitrary convex (preferably sharp) angle pOq and apply the following order:







$\boxed{\text{Example 4.}}$ **Longs *a* and *b* are given . Construct the following longs:**

i) $x = a \cdot b$

ii) $x = \dfrac{a}{b}$

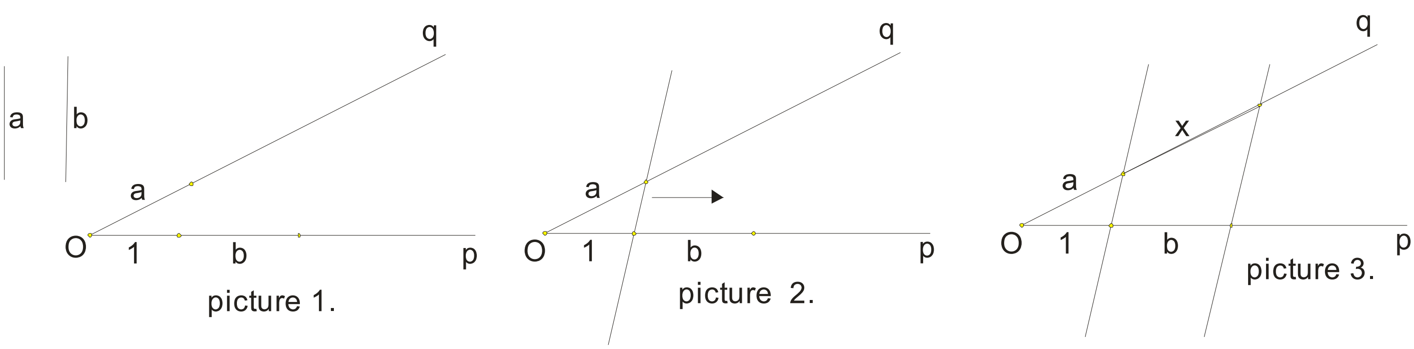
iii) $x = a^2$

<u>Solution</u>

i) $x = a \cdot b$

From here, we have to make a proportion, but so that x is in the last place

$$x = a \cdot b$$
$$1 \cdot x = a \cdot b$$
$$\boxed{1 : a = b : x}$$

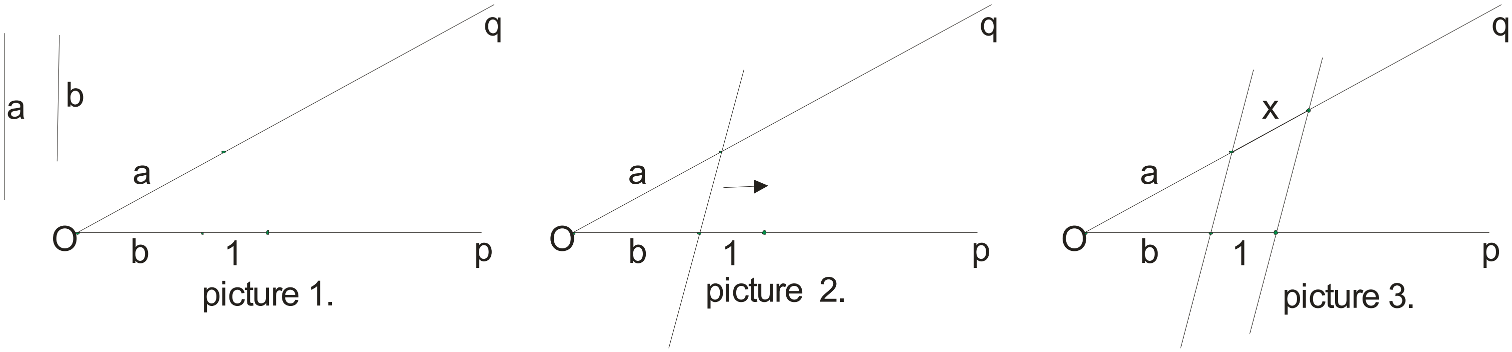


picture 1.   picture 2.   picture 3.

ii) $x = \dfrac{a}{b}$

We have to make a proportion, but so that x is in the last place

$$x = \frac{a}{b}$$
$$\frac{x}{1} = \frac{a}{b} \rightarrow x \cdot b = 1 \cdot a \rightarrow \boxed{b : a = 1 : x}$$

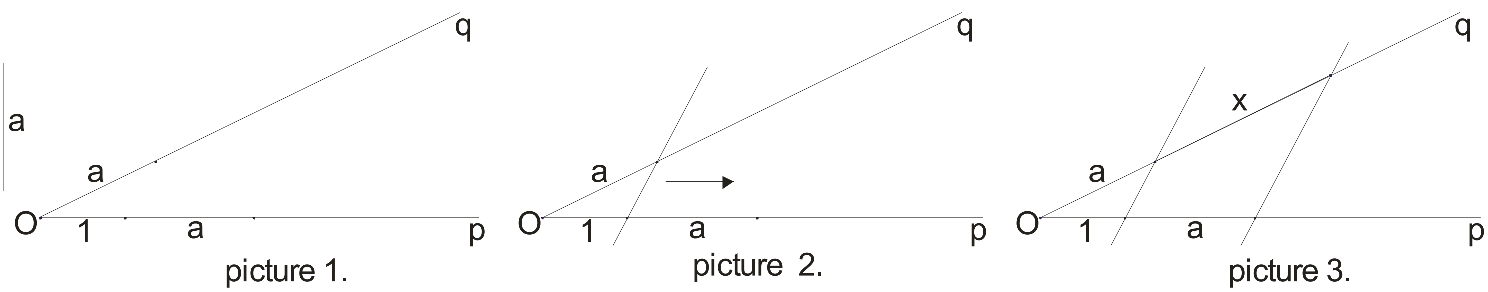


iii) $x = a^2$

$x = a^2$

$1 \cdot x = a \cdot a$

$\boxed{1 : a = a : x}$

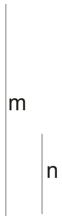


Example 5.

On the line we have points A and B. Determine the point *P* on along *AB* which is shared in the ratio of the two given longer *m* and *n.*

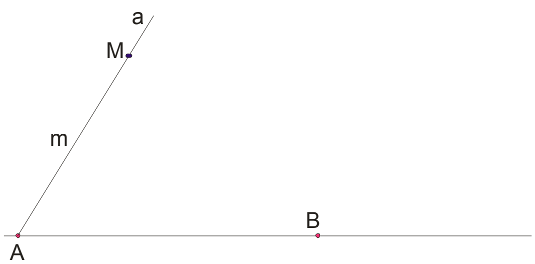
<u>Solution</u>

First choose an arbitrary longs m and n.

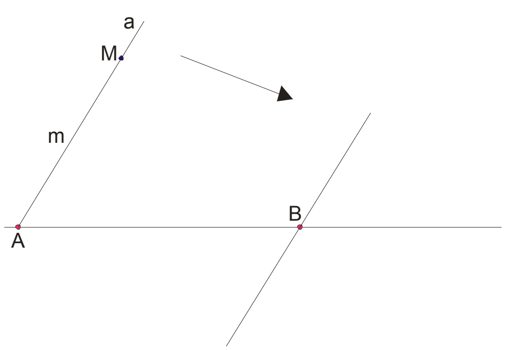


Draw a line with points A and B.

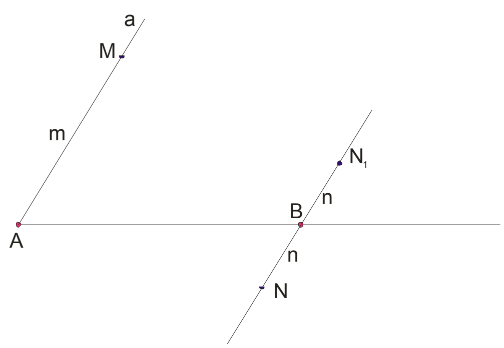
Draw line Aa and m.



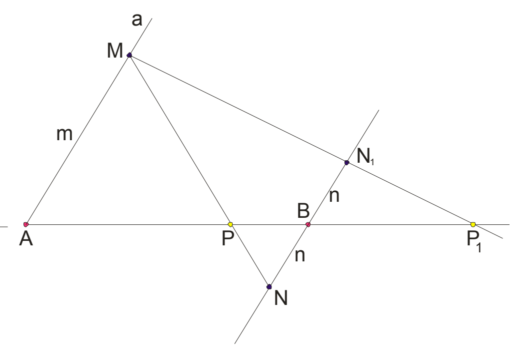
Next pull parallel with Aa through the point B (picture 1.)



picture 1.



picture 2.



picture 3.

On this line we bring n (from point B) on both sides. . We have therefore points N and N_1. (picture 2.)

Merge points N and N_1 with point M and we get section with line AB, that is the points P and P_1.

So we get two solutions and both are good, but it says that a mathematical point P divides AB along the ***inner*** and point P_1 ***external*** division .